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LETTER TO THE EDITOR

A new type of kinetic critical phenomenon

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Abstract. We study a new critical phenomenon in a non-thermal one-dimensional lattice model. It is characterised by the transition from stability to instability of kinks between ordered states. Below the critical point, the kinks are stable and move by annihilating random walks. Above the critical point, they are unstable against creation of kink–antikink pairs. (The *spontaneous* production of pairs is assumed to be absent.) At the critical point $p = p_{cr}$, the density of kinks decreases like $n \sim t^{\alpha}$, with $\alpha = 0.27 \pm 0.08$. Above the critical point, the density of kinks in the stationary state is approximately $(p - p_{cr})^{\beta}$ with $\beta = 0.6 \pm 0.2$. Possible extensions to two or more dimensions and possible applications are discussed.

It is well known that thermal critical phenomena cannot occur in one-dimensional systems. The same is not true for non-thermal systems, provided they have absorbing† (or 'quiescent') states.

The typical example is directed percolation in one space plus one time direction (Durrett 1982, Kinzel and Yeomans 1981), which shows the same critical phenomenon as reggeon field theory (Grassberger and de la Torre 1979, Cardy and Sugar 1980) and the basic contact model (Griffeath 1979). This class of phenomena is characterised by a single absorbing state (all sites not 'wetted'), and thus there is no symmetry breakdown related to them. After realising (Grassberger 1982) that the same critical phenomenon seems to occur also in Schlögl's second model (Schlögl 1972) (contrary to previous investigations which claimed that model to be Ising-like (Nicolis and Malek-Mansour 1980, Brachet and Tirapegui 1981, Borckmans *et al* 1977, 1981)), it was conjectured (Grassberger 1982) that all models with a single absorbing state should show this same critical phenomenon.

In the present letter, we shall study a class of models with two absorbing states. These two states are mutually symmetrical, and thus the transition is accompanied by a spontaneous breakdown of symmetry.

Technically, we shall study two models which are both one-dimensional 'elementary' cellular automata (in the sense of Wolfram (1983)) with very specific added noise. Space and time are discrete in these models. The states are $\{S_i | i \in Z\}$; $S_i = 0, 1$ and the transition rules depend on next neighbours only.

Specifically, the models are chasracterised by the rules

t:	111	101	010	100	001	011 110	000	
$t+1$:	0	0	1	1	1	0 with prob. p 1 with prob. $1-p$	0	model A

† We call a state 'absorbing' if it can be entered but cannot be left. Note that spatially infinite systems can admit several absorbing states in this sense.

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and

t:	111	101	010	100	001	011 110	000	
$t+1$:	0	1	0	1	1	1 with prob. p 0 with prob. $1-p$	0	model B.

For $p=0$, these are rule number 94 (in the notation of Wolfram (1983)) (model A) and 50 (model B). Both of these rules are 'simple' in the sense of Wolfram (1983): when starting with a random initial condition, the system very soon settles in a stationary state (rule 94), resp. in a state of period 2 (rule 50). This is shown also in figure 1.

Consider now very small values of p, i.e. a small probability for $011 \to 0$ in model A and a small probability for $011 \to 1$ in model B. As seen from figure 2, the system now orders itself spontaneously: there are two symmetric absorbing states in both models. For model A, they correspond to a pattern consisting of vertical stripes:

$$S_i = \begin{cases} 0 & i \text{ even} \\ 1 & i \text{ odd} \end{cases} \quad \text{and} \quad S_i = \begin{cases} 1 & i \text{ even} \\ 0 & i \text{ odd} \end{cases}$$

and for model B, they correspond to a chess-board pattern.

After a random start, there are small ordered domains separated by kinks. For $p=0$, these kinks are stationary, but for small $p \neq 0$ they move by annihilating random

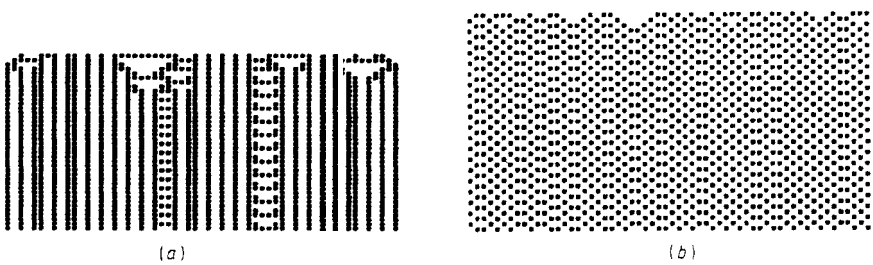

Figure 1. Patterns created, with $p=0$ from (a) model A and (b) model B. The starting configuration was random.

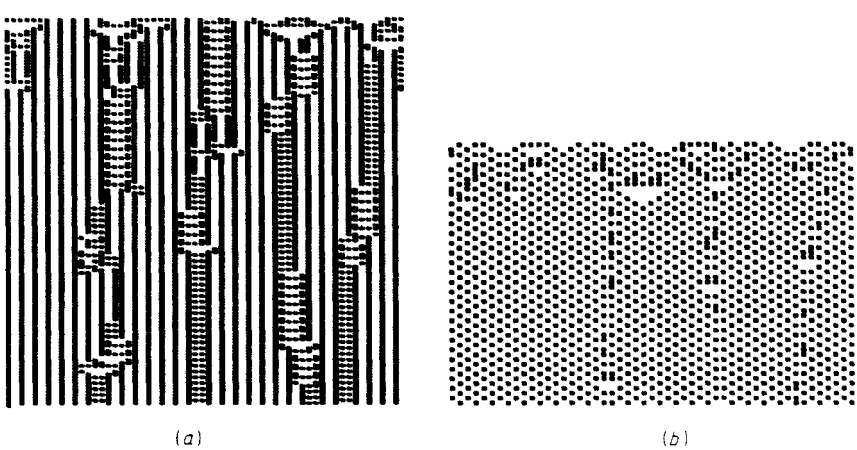

Figure 2. Patterns created, from a random initial configuration, with $p \neq 0$. Again (a) corresponds to model A with $p=0.05$, and (b) corresponds to model B with $P=0.2$.

walks. The diffusion coefficient in both cases is $\propto p$ for small p, and the kink density decreases correspondingly (Griffeath 1979, Grassberger 1983) like $(p \cdot t)^{-1/2}$.

In addition to enhancing the random walk, increasing p has another effect: it leads to a splitting of kinks,

$$\text{kink} \rightarrow \text{kink} + (\text{kink} + \text{antikink}).$$

For small p, this creation of new kinks is outpowered by their annihilation. But above a critical value p_{cr}, a single kink in the initial state is sufficient to create a completely disordered state (see figure 3). For $p = 1$, in particular, model A is just rule 22, while model B is rule 122, in the notation of Wolfram (1983). Both these rules are known to be chaotic (Wolfram 1983, Grassberger 1983).

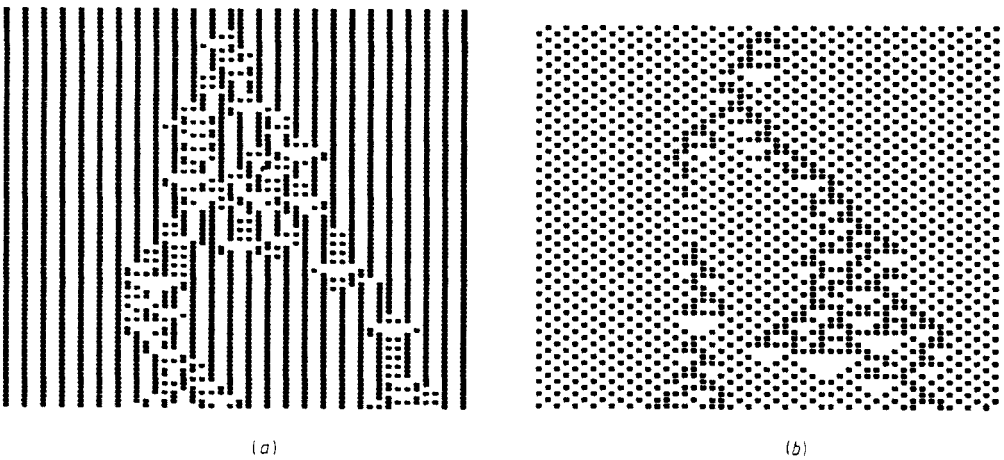

Figure 3. Patterns created from initial states containing a single kink, and with $p > p_{cr}$. (*a*) shows evolution according to model A with $p = 0.25$ and (*b*) shows model B with $p = 0.65$.

In order to study the behaviour near $p = p_{cr}$ more precisely, we have performed detailed Monte Carlo simulations, the results of which are shown in figure 4. They are based on counting the number of doubly occupied neighbours. In the ordered states, both models have no doubly occupied neighbours, thus this number is proportional to the number of kinks. Lattice sizes were 5,000 (model A) and 20,000 (model B) sites. Periodic boundary conditions were chosen in both cases. The critical probability was found to be

$$p_{cr} = \begin{cases} 0.13 \pm 0.02 & \text{model A} \\ 0.555 \pm 0.01 & \text{model B.} \end{cases}$$

At $p = p_{cr}$, in both models the density of doubly occupied neighbours (i.e. the density of kinks) decreases like

$$n_{\text{kink}} \sim t^{-\alpha}, \qquad \alpha = 0.27 \pm 0.08.$$

Above $p = p_{cr}$, the density of kinks in the stationary state goes to zero for $p \rightarrow p_{cr}$ like

$$n_{\text{kink}} \sim (p - p_{cr})^{\beta}, \qquad \beta = 0.6 \pm 0.2.$$

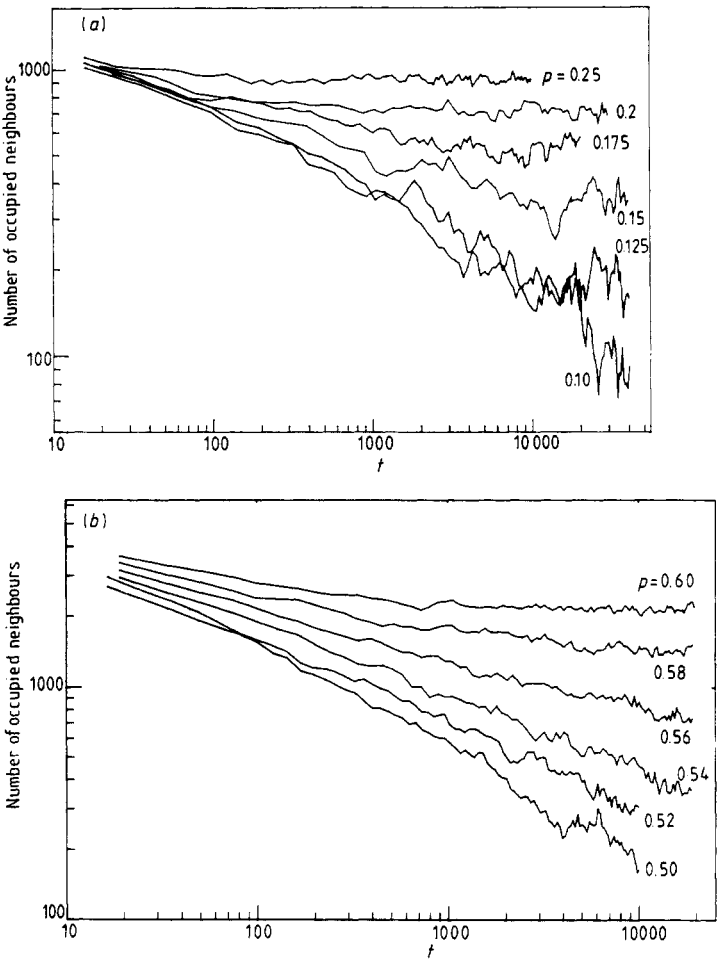

Figure 4. Numbers of occupied neighbouring pairs (measuring the density of kinks), close to the critical probability, as a function of time. For large times, the numbers were averaged over many time steps in order to suppress fluctuations. Lattice sizes were (*a*) 5000 sites (model A) and (*b*) 20 000 sites (model B).

As in other kinetic critical phenomena, there is a third critical exponent measuring the increase of the correlation length when either $t \to \infty$ (for $p = p_{cr}$) or $p \to p_{cr}$ (for $t = \infty$). Our simulations were not precise enough to give a meaningful estimate of this exponent.

Let us now discuss these results.

First, we should point out that the very existence of the transitions depends on the fact that the ordered states were absolutely stable. Even the smallest amount of 'thermal' noise would destroy them. Adding such noise to cellular automata has previously been studied by Wolfram (1983) and Schulman and Seiden (1978).

Secondly, it is not difficult to envisage models with different symmetries, by considering e.g. more than two states per lattice site or with next-nearest neighbour interactions. As a result, one should obtain whole classes of critical phenomena, each with different critical exponents.

When going from one to two dimensions, one can replace the kinks either by point defects or by domain walls. For models with two states per site and NN interactions, the latter is the more natural, as only a domain wall is topologically stable in the same way as a single kink was stable for $0 < p < p_{cr}$ in the above models. Nevertheless, one can artificially construct lattice models in which a single defect can give rise to a defect pair but cannot vanish.

In contrast to the cellular automata studied above, there are automata whose behaviour is extremely complex. The best known example is Conway's game of life (Gardner 1979), but there exist many more similar examples (Farmer and Wolfram 1983). They share with the above rules 94 and 50 the property that they have infinitely many stationary estates. It seems plausible (although we have not yet found an example for such behaviour) that adding *specific* noise to these automata leads to the selection of much less trivially ordered states than in the above models A, B. In the game of life, e.g., extremely complex patterns exist but are very rarely generated from a random start. Adding some transitions with a suitable rate might enhance their production considerably, making the game of life resemble real life much more closely.

Finally, let us suggest a somewhat less speculative application of these ideas. It concerns drawing of single crystals out of a melt. Assume that the original seed contains some defects which prevent ordered growth in their vicinity, leading to randomly moving defects in the plane of growth. On the other hand assume that the melt is sufficiently pure so that spontaneous generation of defects can be neglected. For small drawing velocity, the defects will move until they either annihilate or reach the boundary—leading in this way to a perfect single crystal. For large drawing velocity, any defect might create further defects in the next deposited layer, and formation of a perfect crystal is impossible.

It is of course known that the velocity of drawing has a strong effect on the perfectness of single crystals, but the existence of a sharp transition with associated universal scaling laws has not yet been observed, to our knowledge.

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